MATHEMATICS AND THE DOCTRINE OF FORMAL DISCIPLINE

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The older educators held that discipline acquired in one subject would aid materially in mastering another. Those who defend the opposite view bring evidence to show that knowledge or skill in one thing is little or no preparation for acquiring knowledge or skill in another thing, unless there is an identity of qualities in the two things. A writer in the *School Review* for April, 1905, dealing with this question of formal discipline, undertook to show by experiment that skill in mathematical reasoning is of no value in other kinds of reasoning. His results went to prove the thesis that students' ability to reason in other subjects was inversely proportional to their ability to reason in mathematics. On top of this showing it must be conceded that the actual use to which a knowledge of quantitative relations above the simplest elements of arithmetic is put by most persons is very small. If, then, the mathematics discipline idea is literally and definitively a myth, and skill in perceiving quantitative relations not very important, much of the time now spent on mathematics is probably worse than wasted. Professor Lewis' results, just referred to, ought therefore to challenge the attention of every teacher of secondary and higher mathematics. If this investigator is right, many mathematics teachers ought to be out of a job.

In Professor Lewis' experiments he gave two sets of questions to twenty-four groups of pupils—one of originals in geometry, and the other what might be called originals in general reasoning. A sample of the first was: "Prove that the bisectors of the interior angles of a trapezoid form a quadrilateral two of whose angles are right angles." The others were: "(1) Give all the reasons you can why a high-school education is a good thing. (2) Why should the town rather than the parents pay for the education of its children? (3) Which is of more value, physical or mental training? Give all the reasons you can for the position you take."

After grading the students in each examination, he ranked them in the two kinds of reasoning. The table of ranks given in the article was specially designed to catch the eye and show how those good in mathematics were poor in the other reasoning, and vice versa. His tables can be set down in figures much more compactly and completely in the following way:

TABLE OF RANKS OF PERSONS (p. 290)

Mathematical reasoning	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
General reasoning	13	16	9	5	14	18	11	15	7	6	1	8	12	4	17	3	10	2
Differences in rank	12	14	6	1	9	12	4	7	−2	−4	−10	−4	−1	−10	2	−13	−7	−16

The figures in natural order in the first line represent *individuals* as ranked n mathematics. Thus, No. 1 represents the one best in mathematics, No. 2 the next best, and so on. The figures in the second line designate individuals by their numbers in the first line. Thus, No. 13 in the first line is first in general reasoning, No. 16 of the first line is second in general reasoning, and so on. The third-line figures are obtained by subtracting the figures in the first line from those underneath them. *The third-line figures in this way give the number of points of superiority in general reasoning over mathematical reasoning of individuals arranged in general reasoning order, the best first.* Thus the first in general reasoning was 12 points below this, or thirteenth in mathematical reasoning, the second in general reasoning was 14 points below this rank in mathematical reasoning, or sixteenth; and so on. The negative numbers give the number of points of superiority in mathematical over general reasoning. Thus the ninth in general reasoning was 2 points better, or seventh, in mathematical reasoning. Now, for present purposes, since we do not care for the standings of individuals, the third line tells all we want to know. We will therefore give the tables to follow in this form. Had Professor Lewis chosen to use this form, he might have put down all of his thirty-four tables in the same space he took for the two he gave.

The striking thing about the third-line figures is that they are so large. Had these eighteen students been arranged in general reasoning in exactly the reverse order to that of their mathematical ranking, giving a maximum of differences in ranks, the sum of the figures in the third line would have been 162. Actually the sum was 134. Now, the writer's observation has always been that, while some students were good in mathematics and poor in other subjects, there was a large percentage who were either good in mathematics and good in other subjects, fair in mathematics and fair in other subjects, or poor in mathematics and poor in other subjects. He determined therefore to make a little study on his own account of this question.

In his first and second tests he asked eight instructors in the same school to rank in percentages, *in reasoning power only*, certain students. The average of these markings was taken, and the students were ranked in their ability to carry on miscellaneous kinds of reasoning. The same students were then ranked by taking their marks in mathematics as set down in the records of the school. (It should be added here that originals form a considerable part of the work and of the examination in geometry.) The first set of students ranked were of the maturity of sophomores in college, and the second set of senior preparatory students. Still another table was prepared, after the fashion of that formed from Dartmouth College grades, by contrasting mathematics grades with the average in United States history and commercial geography. These latter subjects were chosen because the burden in the teaching of each was laid not so much on the facts as on seeing the relations between the facts studied. The teachers of these subjects thought that fully 75 per cent. of the work consisted in what was this kind of reasoning.

We will give now Professor Lewis' results and those just referred to in the form already described—i. e., by differences in ranks. The tables numbered with Roman numerals are described immediately below.

TABLES OF RELATIVE RANKS IN VARIOUS TESTS

(The figures are the remainders obtained by subtracting rank in general reasoning from rank in mathematics of students arranged in general reasoning ability, the best first.)

																			Sum	Max.	%
I	12	14	6	1	9	12	4	7	−2	−4	−10	−4	−1	−10	2	−13	−7	−16	134	162	83
II	15	1	11	11	7	13	11	3	11	−9	2	−5	−3	−10	−6	−11	−15	5			
													3	4	−15	−1	−15	−7	194	288	67
III	1	7	11	8	0	4	−1	−4	−8	3	−8	−4	−2	3	−8	0	−2		74	144	51
IV	1	3	8	10	1	2	0	−5	−8	3	−5	3	−3	3	−3	0	−8		66	144	46
V	0	11	7	4	13	−1	0	−6	3	−7	−5	−3	1	3	0	−12	−6	−2	84	162	52
VI	0	10	6	7	13	1	−3	−6	6	−5	−5	−4	−3	0	1	−13	−4	−1	88	162	54
VII	1	1	11	−3	−1	3	4	15	12	7	−3	−7	5	−4	5	−1	2	−5			
							11	9	6	−6	−1	−18	−1	−19	−2	−14	−1	−4	182	450	40

In comparing these tables, let us say that the normal thing would be for persons to be approximately equally good in both tests.

TABLE I, Professor Lewis' first table, was explained above. It shows that 11 persons out of 18, or 61 per cent., were 6 or more points apart in the ranking, while 7 persons, or 39 per cent., were 5 points or less apart. Thus this table shows 61 per cent. abnormal and only 39 per cent. normal.

TABLE II gives the displacements of the Dartmouth College ranks in certain law courses which required reasoning and mathematics. These figures show that 14 persons out of 24, or 58 per cent., were 7 points or more apart in the ranking, while 10 persons, or 42 per cent., were 5 points or less apart. Thus in this experiment 58 per cent. were abnormal and only 42 per cent. normal.

Table III gives the results where the grades in mathematics of students of the maturity of sophomores were contrasted with their reasoning ability in other subjects as estimated by their instructors. It shows that 11 out of 17, or 65 per cent., were 5 points or less apart in the rankings, and 6, or 35 per cent., were 7 points or more apart. In other words, 65 per cent. were normal and 35 per cent. were abnormal. Notice that only 1 was more than 8 points apart when 16 points were possible.

TABLE IV contrasts the geometry and trigonometry grades of the students of Table III with the average from the other instructors. It shows that 13 persons, or 76 per cent., were only 5 or less points apart, and 4 persons, or 24 per cent., 8 points or more apart. That is to say, 76 per cent. were noticeably normal, and only 24 per cent. abnormal. Note the differences in the figures in Tables III and IV, while the percentages are about the same.

TABLE V gives differences in rank between the students of the preparatory set in mathematics and general reasoning. We see that 11 out of 18, or 61 per cent., differ by 5 points or less, while 3, or 17 per cent., differ by 11 points or more. Thus, 61 per cent. are normal and 17 per cent. are badly abnormal.

TABLE VI gives differences of rank of the same set of students as Table V between geometry alone and the general-reasoning average. In this table also 61 per cent. are normal and 17 per cent. abnormal.

TABLE VII gives differences of ranks of a set of students' mathematical standings and their averages in United States history and commercial geography. Here 20 out of 30, or 67 per cent., differ by 6 points or less, and 7, or 23 per cent., differ by 10 points or more. Thus, 67 per cent. are normal and 23 per cent. badly abnormal.

On comparing Professor Lewis' tables with the others, one sees immediately a marked difference. His sums are high as compared with the maximum, while the other sums are comparatively low. Indeed, the results are almost exactly opposite. The class that was abnormal in Professor Lewis' tables becomes the normal one in the other. The latter tables show that, with the exception of about 20 to 25 per cent. of erratic people, those good in mathematics are good in other subjects, those of average ability in mathematics are of average ability in other subjects, and those poor in mathematics are poor in other subjects.

Such contradictory results raise the question of how it could have happened. Various explanations might be advanced. One is that the sharply drawn examination test and the classroom test would be likely to give different results. Doubtless this would make some difference, but it is not very satisfactory as an explanation, since every teacher of mathematics knows that, as a general rule, those good in term work will be good in an examination, no matter whether the examination contain originals or regular propositions. Some other explanation seems necessary.

Professor Lewis says: "Precisely this striking result [viz., ability to reason on practical things varies inversely as ability to reason in mathematics] is discovered in the other twenty-three tests." Having watched the operation of the great law of chances in the happening of events, the writer finds it hard to believe this to have happened without there being some explanation other than the one assigned. Possibly the rankings in the other twenty-three groups were examined only casually. If so, it would be easy to conclude that they were similar, whereas they might be really quite different. A total of 134 in a maximum total of 162 is a large number to have happened in each set. The writer does not, of course, challenge the figures given by Professor Lewis in any way, but thinks there may be some explanation to account for what seems to him remarkable coincidences in the results. It is barely possible that the examiners were prejudiced against certain forms of argument in the general reasoning which those mathematically inclined would be likely to employ.

However. what seems to the writer a much more plausible and satisfactory explanation is a difference in courses. The best preparation for writing on the general-reasoning questions given above would not be a course in mathematics, but a current or recent course in theme-writing. If, in addition to this, one set of students had had a recent course in some subject which would equip them with data concerning general educational questions, they would have a great advantage over others. A moment's reflection will convince anyone that the general effect on an examiner's mind from reading a paper which tested for practical reasoning would depend vastly more on the student's knowledge of data concerning the topic than on his ability to draw formal conclusions correctly. This being true, one could not feel very well satisfied with any conclusions drawn, unless he knew details of the education of the students examined, especially as regards differences of courses or opportunities of general culture. Thus, what

seems at first blush the best test possible by which to judge students as regards their reasoning powers may turn out to be a very poor one. Every experimenter in physical science knows it is often failure to consider some unsuspected detail that vitiates entirely what promised to be important results.

It can be urged, on the other hand, that the tests in reasoning in a variety of subjects for a considerable period of time are likely to furnish a better means of judging students' reasoning power than two diverse and highly specialized examinations. Examining students on a variety of subjects with the materials furnished, by letting them take a course in a subject ought to be a fair test. A very large number of students have been tested in this manner, the results being given in Thorndike's *Educational Psychology*, chapter iv. These results show that mathematics is very often closer to other studies (coefficient of correlation greater) than they are to one another; and, as a rule, if a student is above the average in mathematics, he will be above the average in his other studies.

Out of this discussion comes back to us, then, the question: Does the training in mathematics aid in other reasoning and in the affairs of life? The world in general, and the educational world in particular, says it does, and lays down extended courses in mathematics in all schools. But may not this be a wrong view to take, as Professor Lewis seems to urge? When a student is good in mathematics and good in general reasoning, which is cause and which is effect? Both, or one, or other, or neither? It was Mr. Dooley—was it not—who said in some connection: "Cause and Effect, are they the same? Yes and no."

The present study has no direct bearing on the "faculty" theory of psychology, but purposes merely to show that a relation does exist between ability in mathematical reasoning and general reasoning. As Professor Lewis points out, the existence of such a relationship does not prove the faculty theory true. The tables of the present article seem to show that what a student does in any subject depends more on native endowment and presentation than on the matter considered. It is the native endowment and acquired skill of the pupil that enables him to digest well and be nourished by any intellectual pabulum set before him.

Mathematics gives a training *sui generis*. In arithmetic the problems correlate at many points with the actual affairs of life. Speaking broadly, the most important effects of the mathematical training are abilities of quite general application: as, holding a number of particulars in the mind at one time; training in sustained reasoning; habit of overcoming difficulties; recognizing the universality of the application of correctly stated laws; perceiving the need of care to secure the accuracy of results required; and so on. These powers have identity of qualities with multitudes of activities in which the individual finds himself engaged in after-life. They thus furnish hooks on which to hang new experiences and conquer new problems. In these ways, perhaps more than in any others, mathematics justifies its place in the course.